Class XI Session 2025-26
Subject - Mathematics
Sample Question Paper - 5

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains 38 questions. All questions are compulsory.
2. This Question paper is divided into five Sections - A, B, C, D and E.
3. In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and Questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21 to 25 are Very Short Answer (VSA)-type questions, carrying 2 marks each.
5. In Section C, Questions no. 26 to 31 are Short Answer (SA)-type questions, carrying 3 marks each.
6. In Section D, Questions no. 32 to 35 are Long Answer (LA)-type questions, carrying 5 marks each.
7. In Section E, Questions no. 36 to 38 are Case study-based questions, carrying 4 marks each.
8. There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and one subpart each in 2 questions of Section E.
9. Use of calculators is not allowed.

Section A

1. The minute hand of a watch is 1.4 cm long. How far does its tip move in 45 minutes? **[1]**

a) 7 cm b) 6.3 cm

c) 6 cm d) 6.6 cm

2. The domain of the function f given by f (x) = $\frac{x^2+2x+1}{x^2-x-6}$ **[1]**

a) R – (-3, - 2) b) R – {-2, 3}

c) R – [3, – 2] d) R – {–3, 2}

3. When tested, the lives (in hours) of 5 bulbs were noted as follow: **[1]**
1357, 1090, 1666, 1494, 1623
The mean deviations (in hours) from their mean is

a) 179 b) 356

c) 220 d) 178

4. $\lim\limits_{x \to 0} \frac{\sin x}{x(1+\cos x)}$ is equal to **[1]**

a) $\frac{1}{2}$ b) 1

c) -1 d) 0

5. The distance of the point P (1, – 3) from the line 2y – 3x = 4 is **[1]**

a) $3\sqrt{13}$
b) 13
c) $\sqrt{13}$
d) $\frac{7}{13}\sqrt{13}$

6. A plane is parallel to yz-plane so it is perpendicular to: **[1]**

a) z-axis
b) none of these
c) y-axis
d) x-axis

7. The amplitude of $\frac{1+i\sqrt{3}}{\sqrt{3}+i}$ is **[1]**

a) $-\frac{\pi}{6}$
b) $\frac{\pi}{6}$
c) $\frac{\pi}{3}$
d) $-\frac{\pi}{3}$

8. The number of all 4 digited numbers that can be formed by using the digits 1, 2, 3, 4 and 5 which are divisible by 4 is **[1]**

a) 120
b) 95
c) 12
d) 125

9. $\lim\limits_{x \to a} \frac{x^n - a^n}{x - a}$ is equal to: **[1]**

a) na^{n-1}
b) 1
c) na
d) na^n

10. If tan θ + cot θ = 2, then sin θ = ? **[1]**

a) $\frac{1}{\sqrt{2}}$
b) $\frac{1}{\sqrt{3}}$
c) $\frac{\sqrt{3}}{2}$
d) $\frac{1}{2}$

11. If A = {(x, y) : $x^2 + y^2 = 25$} and B = {(x, y) : $x^2 + 9y^2 = 144$} then A ∩ B contains **[1]**

a) two points
b) one point
c) three points
d) four points

12. If rth term in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{12}$ is without x, then r is equal to **[1]**

a) 8
b) 9
c) 7
d) 10

13. In the expansion of $(1 + x)^{11}$, the 5^{th} term is 24 times the 3^{rd} term. The value of x is **[1]**

a) ± 2
b) ± 3
c) ± 4
d) ± 5

14. Solve the system of inequalities $4x + 3 \geq 2x + 17$, $3x - 5 < -2$, for the values of x, then **[1]**

a) no solution
b) $(-4, 12)$
c) $\left(-\frac{3}{2}, \frac{2}{5}\right)$
d) (-2, 2)

15. For two sets A ∪ B = A if **[1]**

a) $B \subseteq A$
b) A = B
c) $A \subseteq B$
d) $A \neq B$

16. Let A = {1, 2, 3}, B = {2, 3, 4}, then which of the following is a function from A to B ? **[1]**

a) {(1,3), (2,2), (3,3)}

b) {(1,2), (2,3), (3,2), (3,4)}

c) {(1, 3), (1, 4)}

d) {(1,2), (1,3), (2,3), (3,3)}

17. The range of the function f(x) = |x - 1| is **[1]**

a) $[0, \infty)$

b) $(0, \infty)$

c) $(-\infty, 0)$

d) R

18. If C (n, 12) = C (n, 8), then C (22, n) is equal to **[1]**

a) 252

b) 231

c) 303

d) 210

19. **Assertion (A):** If f : R $\rightarrow$ R be defined by f (x) = $\frac{1}{x}$ $\forall x \in R$. Then f is not defined. **[1]**

Reason (R): This function does not exist for every value of the domain.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** If the sum of first two terms of an infinite GP is 5 and each term is three times the sum of the succeeding terms, then the common ratio is $\frac{1}{4}$. **[1]**

Reason (R): In an AP 3, 6, 9, 12 .... the 10th term is equal to 33.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. Express the functions as sets of ordered pairs and determine their ranges: g : A $\rightarrow$ N, g (x) = 2x, where A = [x: x $\in$ N, x $\leq$ 10}. **[2]**

OR

Let A = {1, 2, 3, 4, 5} and B = {1, 4, 5}. Let R be a relation 'is less than' from A to B.

i. List the elements of R.
ii. Find the domain, co-domain and range of R
iii. Depict the above relation by an arrow diagram.

22. Differentiate the function with respect to x: $(3 - 4x)^5$. **[2]**

23. Two dice are thrown simultaneously. Find the probability of getting a prime number as the sum. **[2]**

OR

What is the probability that in a group of two people, both will have the same birthday, assuming that there are 365 days in a year and no one has his/her birthday on 29th February?

24. Find AΔB, if A = {1, 3, 4} and B = {2, 5, 9, 11}. **[2]**

25. Write the distance of the point P (2, 3, 5) from the xy-plane. **[2]**

Section C

26. let f : $[0, \infty) \rightarrow R$ and g : $R \rightarrow R$ be defined by $f(x) = \sqrt{x}$ and g(x) = x. Find f + g, f - g, fg, and g/g. **[3]**

27. Find the equations of two straight lines passing through (1, 2) and making an angle of 60° with the line x + y = 0. Find also the area of the triangle formed by the three lines. **[3]**

[3]

28. Find the coefficients of x^{32} and x^{-17} in the expansion of $\left(x^4-\frac{1}{x^3}\right)^{15}$

OR

Show that the middle term in the expansion of $\left(x-\frac{1}{x}\right)^{2n}$ is $\frac{1\cdot3\cdot5...(2n-1)}{n!}(-2)^n$

29. Differentiate the function sin x^2 with respect to x from first principle. **[3]**

OR

Evaluate: $\lim\limits_{x\to0}\frac{\cos 2x-1}{\cos x-1}$

30. Evaluate: $\sum\limits_{k=1}^{11}\left(2+3^k\right)$ **[3]**

OR

Three years before the population of a village was 10000. If at the end of each year, 20% of the people migrated to a nearby town, what is its present population?

31. The letters of the word **RANDOM** are written in all possible orders and these words are written out as in a dictionary. Find the rank of the word **RANDOM**. **[3]**

Section D

32. Find the variance and standard deviation for the following distribution. **[5]**

x_i	4.5	14.5	24.5	34.5	44.5	54.5	64.5
f_i	1	5	12	22	17	9	4

33. Find the equation of a circle passing through the point (7, 3) having radius 3 units and whose centre lies on the line y = x – 1. **[5]**

OR

Find the equation of the circle which passes through the centre of the circle $x^2 + y^2 + 8x + 10y - 7 = 0$ and is concentric with the circle $2x^2 + 2y^2 - 8x - 12y - 9 = 0$

34. Solve the following system of linear inequalities **[5]**

$-2-\frac{x}{4}\geq\frac{1+x}{3}$ and 3 - x < 4(x-3)

35. $0\leq x\leq\pi$ and x lies in the IInd quadrant such that sin x $=\frac{1}{4}$. Find the values of $\cos\frac{x}{2}$, $\sin\frac{x}{2}$ and $\tan\frac{x}{2}$. **[5]**

OR

Prove that: cot x + $\cot\left(\frac{\pi}{3}+x\right)$ + $\cot\left(\frac{2\pi}{3}+x\right)$ = 3 cot 3x.

Section E

36. **Read the following text carefully and answer the questions that follow:** **[4]**

A sequence of non-zero numbers is said to be a geometric progression, if the ratio of each term, except the first one, by its preceding term is always constant.

Rahul being a plant lover decides to open a nursery and he bought a few plants with pots. He wants to place pots in such a way that the number of pots in the first row is 2, in the second row is 4 and in the third row is 8 and so on ... .

i. Represent the above information in form of sequence name the sequence and also find the constant multiple by which the number of pots is increasing in every row? (1)

ii. If Rahul wants to place 510 pots in total, find then the total number of rows formed in this arrangement? (1)

iii. Find the total number of pots up to 10^{th} row? (2)

OR

Find the total number of pots up to 8^{th} row? (2)

37. **Read the following text carefully and answer the questions that follow:** **[4]**

Four friends Dinesh, Yuvraj, Sonu, and Rajeev are playing cards. Dinesh, shuffling a cards and told to Rajeev choose any four cards.

i. What is the probability that Rajeev getting all face card. (1)

ii. What is the probability that Rajeev getting two red cards and two black card. (1)

iii. What is the probability that Rajeev getting one card from each suit. (2)

OR

What is the probability that Rajeev getting two king and two Jack cards. (2)

38. **Read the following text carefully and answer the questions that follow:** **[4]**

A class teacher Mamta Sharma of class XI write three sets A, B and C are such that A = {1, 3, 5, 7, 9}, B = {2, 4, 6, 8} and C = {2, 3, 5, 7, 11}.

i. Write the intersecting of two set A and C? (1)

ii. Write the condition for two sets A and B to be disjoint? (1)

iii. Find A ∩ C. (2)

OR

Find A ∩ B. (2)

Solution

Section A

1.

(d) 6.6 cm

Explanation:

Angle traced by the minute hand in 60 min $= (2\pi)^c$

Angle traced by the minute hand in 45 min $= \left(\frac{2\pi}{60} \times 45\right)^c = \left(\frac{3\pi}{2}\right)^r$

$\therefore$ r = 1.4 cm and $\theta = \left(\frac{3\pi}{2}\right)^c$

$\Rightarrow l = r\theta = \left(1.4 \times \frac{3\pi}{2}\right)$ cm $= \left(1.4 \times \frac{3}{2} \times \frac{22}{7}\right)$ cm = 6.6 cm

2.

(b) R – {-2, 3}

Explanation:

We have, f(x) $= \frac{x^2+2x+1}{x^2-x-6}$

f(x) is not defined, if $x^2 - x - 6 = 0$

$\Rightarrow$ (x - 3)(x + 2) = 0

$\therefore$ x = -2, 3

$\therefore$ Domain of f = R - {-2, 3}

3.

(d) 178

Explanation:

Given the lives (in hours) of 5 bulbs is 1357, 1090, 1666, 1494, 1623

Here mean, $\overline{x} = \frac{1357+1090+1666+1494+1623}{5} = \frac{7230}{7}$ = 1446

This can be written in table form as,

Lives (in hours)(x_i)	$d_i = \lvert x_i - \bar{x}\rvert$
1357	= |1357 - 1446| = 89
1090	= |1090 - 1446| = 356
1666	= |1666 - 1446| = 220
1494	= |1494 - 1446| = 177
1623	= |1623 - 1446| = 177
Tootal	$\sum d_i$ = 890

Hence Mean Deviation becomes,

M.D = $\frac{\sum d_i}{n} = \frac{890}{5}$ = 178

Therefore, the mean deviation about the mean of the lives of 5 bulbs is 178

4. **(a)** $\frac{1}{2}$

Explanation:

We have

$$\lim_{x\to0} \frac{\sin x}{x(1+\cos x)} = \lim_{x\to0} \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{x \quad 2\cos^2\frac{x}{2}}$$

$$= \frac{1}{2}\lim_{x\to0} \frac{\tan\frac{x}{2}}{\frac{x}{2}} = \frac{1}{2}$$

5.

(c) $\sqrt{13}$

Explanation:

We know that the distance of the point P (1, – 3) from the line 2y – 3 x – 4 = 0 is the length of perpendicular from the point to the line which is given by

$\left|\frac{2(-3)-3-4}{\sqrt{13}}\right|=\sqrt{13}$

6.

(d) x-axis

Explanation:

Any plane parallel to yz-plane is perpendicular to x-axis.

7.

(b) $\frac{\pi}{6}$

Explanation:

$\frac{\pi}{6}$

Let z $=\frac{1+i\sqrt{3}}{\sqrt{3}+i}$

$\Rightarrow z=\frac{1+i\sqrt{3}}{\sqrt{3}+i}\times\frac{\sqrt{3}-i}{\sqrt{3}-i}$

$\Rightarrow z=\frac{\sqrt{3}+2i-\sqrt{3}i^2}{3-i^2}$

$\Rightarrow z=\frac{\sqrt{3}+\sqrt{3}+2i}{4}$

$\Rightarrow z=\frac{2\sqrt{3}+2i}{4}$

$\Rightarrow z=\frac{\sqrt{3}}{2}+\frac{1}{2}i$

$\tan\alpha=\left|\frac{\operatorname{Im}(z)}{\operatorname{Re}(z)}\right|$

$=\frac{1}{\sqrt{3}}$

$\Rightarrow \alpha=\frac{\pi}{6}$

since, z lies in the first quadrant.

Therefore, arg (z) $=\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)=\frac{\pi}{6}$

8.

(d) 125

Explanation:

The number divisible by 4 must have last two-digit divided by four

So number formed can be any of the following four types

**12

**24

**32

**44

**52

Since repetition is allowed in **12, the first place can be filled in 5 ways, the second place also can be filled in 5 ways

Hence, number of numbers ending with 12 = 5 $\times$ 5 = 25

Similarly, numbers ending with 24, 32, 44, 52 each will be = 5 $\times$ 5 = 25

Hence the number of all 4 digited numbers that can be formed by using the digits 1, 2, 3, 4 and 5 which are divisible by 4 = 25 $\times$ 5 = 125 [since there are 5 such cases]

9. **(a)** na^{n-1}

Explanation:

$\lim\limits_{x\to a}\frac{x^n-a^n}{x-a}$

$=\lim\limits_{x\to a^+}\frac{x^n-a^n}{x-a}$ [$\because$ f(x) exists, $\lim\limits_{x\to a}$ f(x) = $\lim\limits_{x\to a^+}$ f(x)]

$= \lim_{h\to 0} \frac{(a+h)^n - a^n}{a+h-a}$

$= \lim_{h\to 0} a^n \frac{\left[\left(1+\frac{h}{a}\right)^n - 1\right]}{h}$

$= a^n \lim_{h\to 0} [1 + n \cdot \frac{h}{a} + \frac{n(n-1)}{2!}\frac{h^2}{a^2} ... + ... -1]$

$= a^n \lim_{h\to 0} [\frac{n}{a} + \frac{h(h-1)}{2!}\frac{h}{a^2} + ...]$

$= a^n \frac{n}{a}$

$= na^{n-1}$

10. **(a)** $\frac{1}{\sqrt{2}}$

Explanation:

$\tan\theta + \cot\theta = 2 \Rightarrow \tan\theta + \frac{1}{\tan\theta} = 2$

$\Rightarrow \tan^2\theta + 1 = 2\tan\theta \Rightarrow 1 + \tan^2\theta - 2\tan\theta = 0$

$\Rightarrow (1-\tan\theta)^2 = 0 \Rightarrow 1 - \tan\theta = 0 \Rightarrow \tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$

$\therefore \sin\theta = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

11.

(d) four points

Explanation:

From A, $x^2 + y^2 = 25$ and from B, $x^2 + 9y^2 = 144$

$\therefore$ From B, $(x^2 + y^2) + 8y^2 = 144$

$\Rightarrow 25 + 8y^2 = 144$

$\Rightarrow 8y^2 = 119$

$\Rightarrow y = \pm\sqrt{\frac{119}{8}}$

$\therefore x^2 + y^2 = 25 \Rightarrow x^2 = 25 - y^2 = 25 - \frac{119}{8} = \frac{81}{8}$

$\Rightarrow x = \pm\sqrt{\frac{81}{8}}$

Since we solved equations simultaneously, therefore A ∩ B has four points A has 2 elements & B has 2 elements.

12.

(b) 9

Explanation:

rth term in the given expansion is ${}^{12}C_{r-1}\,(2x^2)^{12-r+1}\left(\frac{-1}{x}\right)^{r-1}$

$= (-1)^{r-1}\,{}^{12}C_{r-1}\,2^{13-r}\,x^{26-2r-r+1}$

For this term to be independent of x, we must have:

$27 - 3r = 0$

$\Rightarrow r = 9$

Therefore, the required 9th term in the expansion is independent of x.

13. **(a)** ± 2

Explanation:

We have the general term of $(1 + x)^n$ is $T_{r+1} = {}^nC_r(x)^r$

Consider $(1 + x)^{11}$

Hence general term $T_{r+1} = {}^{11}C_r(x)^r$

Given $T_5 = 24 \times T_3$

$\Rightarrow {}^{11}C_4(x)^4 = 24 \times {}^{11}C_2(x)^2$

$\Rightarrow \frac{24\times{}^{11}C_2}{{}^{11}C_4} = (x)^2$

$\Rightarrow \frac{24\times 55}{330} = (x)^2$

$\Rightarrow 4 = (x)^2$

$\Rightarrow x = \pm 2$

14. **(a)** no solution

Explanation:

We have given: $4x + 3 \geq 2x + 17$

$\Rightarrow 4x - 2x \geq 17 - 3 \Rightarrow 2x \geq 1$

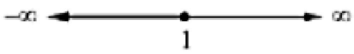
$\Rightarrow x \geq \frac{14}{2}$ [Dividing by 2 on both sides]

$\Rightarrow x \geq 7$ ..... (i)

Also we have 3x - 5 < -2

$\Rightarrow \quad 3x < -2 + 5 \Rightarrow 3x < 3$

$\Rightarrow \quad x < 1$



On combining (i) and (ii), we see that solution is not possible because nothing is common between these two solutions.(i.e., $x < 1$, $x \geq 7$)

15. **(a)** $B \subseteq A$

Explanation:

The union of two sets is a set of all those elements that belong to A or to B or to both A and B.

If $A \cup B = A$, then $B \subseteq A$

16. **(a)** {(1,3), (2,2), (3,3)}

Explanation:

A relation is a function if first entry in each pair (element) is not repeated.

17. **(a)** $[0, \infty)$

Explanation:

A modulus function always gives a positive value

$R(f) = [0, \infty)$

18.

(b) 231

Explanation:

${}^nC_{12} = {}^nC_8$

$\Rightarrow$ n = 12 + 8 = 20 [$\because {}^nC_x = {}^nC_y \Rightarrow$ n = x + y or x = y]

Now,

${}^{22}C_n = {}^{22}C_{20}$

$= \frac{22}{2} \times \frac{21}{1}$

= 231.

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation:

Both assertion and reason are true because in the given function at x = 0, $f(x) = \frac{1}{0} = \infty$. So, the function is not define

20.

(c) A is true but R is false.

Explanation:

Assertion Let a be the first term and r(|r| < 1) be the common ratio of the GP.

$\therefore$ The GP is a, ar, ar^2,...

According to the question,

$T_1 + T_2 = 5 \Rightarrow a + ar = 5 \Rightarrow a(1 + r) = 5$

and $T_n = 3(T_{n+1} + T_{n+2} + T_{n+3} + ...)$

$\Rightarrow ar^{n-1} = 3(ar^n + ar^{n+1} + ar^{n+2} + ...)$

$\Rightarrow ar^{n-1} = 3ar^n(1 + r + r^2 + ...)$

$\Rightarrow 1 = 3r(\frac{1}{1-r})$

$\Rightarrow$ 1 - r = 3r

$\Rightarrow r = \frac{1}{4}$

Reason: Given, 3, 6, 9, 12 ...

Here, a = 3, d = 6 - 3 = 3

$\therefore T_{10}$ = a + (10 - 1)d

= 3 + 9 $\times$ 3

= 3 + 27 = 30

Section B

21. We have, g(x) = 2x and A = {1, 2, 3, ...10). Therefore,

g(1) = 2 $\times$1 = 2, g(2) = 2 $\times$ 2 = 4, g(3) = 2 $\times$ 3 = 6, g(4) = 2 $\times$ 4 = 8, g(5) = 2 $\times$ 5 = 10,

g(6) = 2 $\times$ 6 = 12, g(7) = 2 $\times$ 7 = 14, g(8) = 2 $\times$ 8 = 16, g(9) = 2 $\times$ 9 = 18

and, g(10) = 2 $\times$ 10 = 20.

$\therefore$g = {(x, g(x) : x $\in$ A} = {(1, 2), (2, 4), (3, 6),...,(10, 20)}.

Hence, Range of g = g(A) = {g(x) : x $\in$ A} = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}.

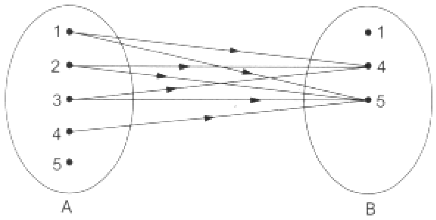
OR

Here, A = {1, 2, 3, 4, 5}, B = {1, 4, 5} and R is a relation 'is less than' from A to B.

i. R = {(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5), (4, 5)}

ii. Dom (R) = {1, 2, 3, 4), range (R) = {4, 5} and co-domain (R) = {1, 4, 5}.

iii. We may represent the above relation by an arrow diagram, shown below.



22. To Find: $\frac{d}{dx}(3-4x)^5$

Formula used : $\frac{d}{dx}(y^n) = ny^{n-1} \times \frac{dy}{dx}$

Let y = $(3 - 4x)^5$

So by using the above formula, we have

$\frac{d}{dx}(3- 4x)^5 = 5(3 - 4x)^4 \times \frac{d}{dx}$ (3 -4x) = - 20 $(3 - 4x)^4$

Differentiation of y = $(3 -4x)^5$ is $-20(3 - 4x)^4$

23. We know that in a single throw of two dice, the total number of possible outcomes is (6 $\times$ 6) = 36.

Let S be n(the sample space of the event and is given by

S) = 36.

Let E_3 = event of getting a prime number as the sum. Then,

E_3 = {(1,1), (1,2), (1,4), (1,6), (2,1), (2,3), (2,5), (3,2), (3,4), (4,1), (4,3), (5,2), (5,6), (6,1), (6, 5)}

$\Rightarrow n(E_3) = 15$

$\therefore P(E_3) = \frac{n(E_3)}{n(S)} = \frac{15}{36} = \frac{5}{12}$

OR

We know that,

Probability of occurring = 1 - the probability of not occurring

So we have to find the probability of not occurring, i.e. probability such that both of them don't have a birthday on the same day.

Let us suppose the first person has a birthday on a particular day then the other person can have a birthday in the remaining 364 days

The Probability of not having the same birthday = $\frac{364}{365}$

The Probability of having same birthday = 1 - the probability of not having the same Birthday

= 1 - $\frac{364}{365}$

$= \frac{1}{365}$

Conclusion: Probability of two persons having the same birthday is $\frac{1}{365}$

24. We know that AΔB represents the symmetric difference between sets A and B.

That is, AΔB = (A - B) ∪ (B -A)

According to the Question,

A = {1, 3, 4} and B = {2, 5, 9, 11}

Then, (A - B) = {1, 3, 4} - {2, 5, 9, 11} = {1, 3, 4}

and (B - A) = {2, 5, 9, 11} - {1, 3, 4} = {2, 5, 9, 11}

Hence,

AΔB = (A - B) ∪ (B - A)

= {1, 3, 4} ∪ {2, 5, 9, 11}

= {1, 2, 3, 4, 5, 9, 11}

25. Given: Points P(2, 3, 5)

As we know z = 0 in xy-plane.

The shortest distance of the plane will be the z-coordinate of the point

Hence, the distance of point P from xy-plane is 5 units

Section C

26. According to the question , we can write,

Given $f(x) = \sqrt{x}$ and g(x) = x

Domain of = $[0, \infty)$

Domain of g = R

We know (f + g) (x) = f(x) + g(x)

Domain of f + g = Domain of Domain of g

Domain of f + g = $[0, \infty) \cap R$

Domain of f + g = $[0, \infty)$

Thus, f + g $[0, \infty)$ R is given by

f - g

We know (f – g) (x) = f(x) - g(x)

$\therefore (f - g)(x) = \sqrt{x} - x$

Domain of f - g = Domain of Domain of g

= Domain of f - g = $[0, \infty) \cap R$

Domain of f - g = $[0, \infty)$

Thus, f - g :$[0, \infty) \rightarrow$ R is given by

We know (fg) (x) = f(x) g(x)

$\Rightarrow (fg)(x) = \sqrt{x} \times x$

$\Rightarrow (fg)(x) = x^{\frac{1}{2}} \times x$

$\therefore (fg)(x) = x^{\frac{3}{2}}$

Clearly, (fg) (x) is also defined only for non-negative real numbers x as a square of a real number is never negative.

Thus, fg $[0, \infty) \rightarrow R$ is given by $(fg)(x) = x^{\frac{3}{2}}$

We know $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

$\Rightarrow \left(\frac{f}{g}\right)(x) = \frac{\sqrt{x}}{x}$

$\Rightarrow \left(\frac{\mathrm{f}}{\mathrm{g}}\right)(\mathrm{x}) = \frac{\sqrt{\mathrm{x}}}{(\sqrt{\mathrm{x}})^2}$

$\therefore \left(\frac{\mathrm{f}}{\mathrm{g}}\right)(\mathrm{x}) = \frac{1}{\sqrt{\mathrm{x}}}$

Clearly, $\left(\frac{f}{g}\right)(x)$ is defined for all positive real values of x, except for the case when x = 0.

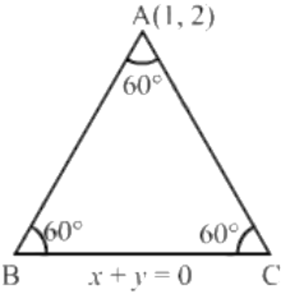
When x = 0, will be undefined as the division result will be indeterminate.

Domain of $\left(\frac{f}{g}\right)(x)$ = $[0, \infty) - \{0\}$

Domain of $\left(\frac{f}{g}\right)(x)$ = $(0, \infty)$

Thus, f/g $(0, \infty) \rightarrow R$ is given by $\left(\frac{f}{g}\right)(x) = \frac{1}{\sqrt{x}}$

27. Suppose A(1, 2) be the vertex of the triangle ABC and x + y = 0 be the equation of BC.



Now, we have to find the equations of sides AB and AC, each of which makes an angle 60° with the line x + y = 0

We know the equations of two lines passing through a point (x_1, y_1) and making an angle α with the line whose slope is m.

$\implies y - y_1 = \frac{m \pm \tan\alpha}{1 \mp m\tan\alpha}(x - x_1)$...(i)

Here,we have

$x_1 = 1, y_1 = 2, \alpha = 60°, m = -1$,substituting in (i)

Therefore, the equations of the required sides are

$\implies y - 2 = \frac{-1+\tan 60°}{1+\tan 60°}(x - 1)$ and $y - 2 = \frac{-1-\tan 60°}{1-\tan 60°}(x - 1)$

$\Rightarrow y - 2 = \frac{\sqrt{3}-1}{\sqrt{3}+1}(x - 1)$ and $y - 2 = \frac{\sqrt{3}+1}{\sqrt{3}-1}(x - 1)$

$\Rightarrow y - 2 = (2 - \sqrt{3})(x - 1)$ and $y - 2 = (2 + \sqrt{3})(x - 1)$

Solving x + y = 0 and $y - 2 = (2 - \sqrt{3})(x - 1)$, we obtain

$x = -\frac{\sqrt{3}+1}{2}, y = \frac{\sqrt{3}+1}{2}$

$\therefore B \equiv \left(-\frac{\sqrt{3}+1}{2}, \frac{\sqrt{3}+1}{2}\right)$ or $C \equiv \left(\frac{\sqrt{3}-1}{2}, -\frac{\sqrt{3}-1}{2}\right)$

AB = BC = AD = $\sqrt{6}$ units

$\therefore$ Area of the required triangles $= \frac{\sqrt{3}\times(\sqrt{6})^2}{4} = \frac{3\sqrt{3}}{2}$ square units.

28. Let general term in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ be T_{r+1}

Then, $T_{r+1} = {}^{15}C_r (x^4)^{15-r} (-\frac{1}{x^3})^r$

$\Rightarrow T_{r+1} = (-1)^r \times {}^{15}C_r (x)^{60-7r}$ ... (i)

This term will contain x^{32}, if 60 - 7r = 32

$\Rightarrow 7r = 60 - 32 = 28 \Rightarrow r = \frac{28}{7} = 4$

$\therefore$ The term containing x^{32} is T_{4+1} i.e., T_5.

Coefficient of $x^{32} = (-1)^4 \times {}^{15}C_4 = 1365$ [using Eq. (i)]

Now, for the term containing x^{-17}, put 60 - 7r = -17

$\Rightarrow 7r = 77 \Rightarrow r = 11$

So, the term containing x^{-17} is T_{11+1} i.e., T_{12}.

$\therefore$ Coefficient of $x^{-17} = (-1)^{11} \times {}^{15}C_{11} = -1365$ [using Eq. (i)]

OR

The exponent in $\left(x - \frac{1}{x}\right)^{2n}$ is an even natural number. So, $\left(\frac{2n}{2} + 1\right)^{th}$ i.e. $(n + 1)^{th}$ terms is the middle term and is given by

$T_{n+1} = {}^{2n}C_n (x)^{2n-n}\left(-\frac{1}{x}\right)^n$

$\Rightarrow \quad T_{n+1} = \frac{(2n)!}{n!n!}x^n \times \frac{(-1)^n}{x^n}$

$\Rightarrow \quad T_{n+1} = \frac{1\cdot2\cdot3\cdot4\cdot5\cdot6\ldots(2n-1)(2n)}{n!n!} \times (-1)^n$

$\Rightarrow \quad T_{n+1} = \frac{\{1\cdot3\cdot5\ldots(2n-1)\}\{2\cdot4\cdot6\ldots(2n-2)(2n)\}}{n!n!} \times (-1)^n$

$\Rightarrow \quad T_{n+1} = \frac{2^n\{1\cdot3\cdot5\ldots(2n-1)\}\{1\cdot2\cdot3\ldots(n-1)n\}}{n!n!} \times (-1)^n$

$\Rightarrow \quad T_{n+1} = \frac{1\cdot3\cdot5\ldots(2n-1)}{n!} \times 2^n \times (-1)^n = \frac{1\cdot3\cdot5\ldots(2n-1)}{n!} \times (-2)^n$

29. Let $f(x) = \sin x^2$. Then, $f(x + h) = \sin(x + h)^2$

$\therefore \frac{d}{dx}(f(x)) = \lim_{h\to0}\frac{f(x+h)-f(x)}{h}$

$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to0}\frac{\sin(x+h)^2 - \sin x^2}{h}$

$\Rightarrow \frac{d}{dx}(f(x)) = \lim\limits_{h\to 0}\frac{2\sin\left(\frac{2hx+h^2}{2}\right)\cos\left(\frac{2x^2+2hx+h^2}{2}\right)}{h}$

$\Rightarrow \frac{d}{dx}(f(x)) = \lim\limits_{h\to 0}\frac{2\sin\left(\frac{2hx+h^2}{2}\right)}{h\left(\frac{2x+h}{2}\right)}\left(\frac{2x+h}{2}\right)\cos\left(\frac{2x^2+2hx+h^2}{2}\right)$

$\Rightarrow \frac{d}{dx}(f(x)) = \lim\limits_{h\to 0}\frac{\sin\left(\frac{2hx+h^2}{2}\right)}{\left(\frac{2hx+h^2}{2}\right)}\times\lim\limits_{h\to 0}(2x+h)\times\lim\limits_{h\to 0}\cos\left(\frac{2x^2+2hx+h^2}{2}\right)$

$\Rightarrow \frac{d}{dx}(f(x)) = \lim\limits_{\theta\to 0}\frac{\sin\theta}{\theta}\times\lim\limits_{h\to 0}(2x+h)\times\lim\limits_{h\to 0}\cos\left(\frac{2x^2+2hx+h^2}{2}\right)$

where $\theta = \frac{2hx+h^2}{2}$

$\Rightarrow \frac{d}{dx}(f(x))$ = (1) x (2x) cos x^2 = 2x cos x^2

$\therefore \frac{d}{dx}\left(\sin x^2\right)$ = 2x cos x^2

OR

We have $\lim\limits_{x\to 0}\frac{\cos 2x-1}{\cos x-1}$

At x = 0, the value of the given function takes the form $\frac{0}{0}$

Now,

$= \lim\limits_{x\to 0}\frac{\cos 2x-1}{\cos x-1}$

$= \lim\limits_{x\to 0}\frac{1-2\sin^2 x-1}{1-2\sin^2\frac{x}{2}-1}\left\{\cos 2x = 1-2\sin^2 x\right\}$

$= \lim\limits_{x\to 0}\frac{\sin^2 x}{\sin^2\left(\frac{x}{2}\right)}$

$= \lim\limits_{x\to 0}\frac{\left(\frac{\sin x^2}{x}\right)\times x^2}{\left(\frac{\sin\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)}\right)^2\times\frac{x^2}{4}}$

$= \frac{4\lim\limits_{x\to 0}\frac{(\sin x)^2}{x}}{\lim\limits_{x\to 0}\left(\frac{\sin\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)}\right)^2}\left\{\lim\limits_{x\to 0}\frac{\sin x}{x} = 1\right\}$

$= \frac{4\times 1}{1} = 4$

30. Given: $\sum_{k=1}^{11}\left(2+3^k\right)$

= $(2 + 3^1) + (2 + 3^2) + (2 + 3^3) + (2 + 3^{11})$

= (2 + 2 + 2 +........11 times) + ($3 + 3^2 + 3^3$ +....... $+3^{11}$)

= 22 + ($3 + 3^2 + 3^3$ +....... $+3^{11}$) ……….(i)

Here 3, 3^2,3^3 .... ,$3^{11}$is in G.P.

$\therefore$a = 3 and r = $\frac{3^2}{3} = 3$

$S_n = \frac{3\left(3^{11}-1\right)}{3-1} = \frac{3}{2}\left(3^{11}-1\right)$

Putting the value of S_n in eq. (i), we get $\sum_{k=1}^{11}\left(2+3^k\right) = 22+\frac{3}{2}\left(3^{11}-1\right)$

OR

Here,it is given that: Three years back population = 10000

Time = 3 years

Rate = 20% per annum

We have,Number of people migrated on the very first year is 20% of 10000

$\Rightarrow \frac{10000\times 20}{100}$ = 2000

People left after migration in the very first year = 10000 - 2000 = 8000

Number of people migrated in the second year is 20% of 8000

$\Rightarrow \frac{8000\times 20}{100}$ = 1600

Therefore,People left after migration in the second year = 8000 - 1600 = 6400

Number of people migrated in the third year is 20% of 6400

$\Rightarrow \frac{6400\times 20}{100}$,= 1280

Now, people left after migration in the third year = 6400 - 1280 = 5120

Therefore,the present population is 5120.

31. In a dictionary, the words at each stage are arranged in alphabetical order. In the given problem, we must consider the words beginning with A, D, M, N, O, R in order. A will occur in the first place as often as there are ways of arranging the remaining 5 letters all at a time i.e., A will occur 5! times. Similarly, D, M, N, O will occur in the first place as the same number of times.

$\therefore$ Number of words starting with letter A

= 5! = 120

Number of words starting with letter D

= 5! = 120

Number of words starting with letter M

= 5! = 120

Number of words starting with letter N

= 5! = 120

Number of words starting with letter O

= 5! = 120

Number of words beginning with letter R is 5! but one of these words is the word RANDOM.

So, we first find the number of words beginning with RAD and RAM.

Number of words starting with RAD = 3! = 6

Number of words starting with RAM

= 3! = 6

Now, the words beginning with 'RAN' must follow.

There are 3! words beginning with RAN. One of these words is the word RANDOM itself.

The first word beginning with RAN is the word RANDMO and the next word is RANDOM.

$\therefore$ Rank of RANDOM = 5 $\times$ 120 + 2 $\times$ 6 + 2

= 600 + 12 + 2

= 614

Section D

32. We need to make the following table from the given data:

x_i	f_i	$d_i = x_i - 34.5$	$u_i = \frac{x_i - 34.5}{10}$	u_i^2	f_iu_i	$f_iu_i^2$
4.5	1	-30	-3	9	-3	9
14.5	5	-20	-2	4	-10	20
24.5	12	-10	-1	1	-12	12
34.5	22	0	0	0	0	0
44.5	17	10	1	1	17	17
54.5	9	20	2	4	18	36
64.5	4	30	3	9	12	36
Total	N = 70				22	130

The formula to calculate the Variance is given as, $\sigma^2 = \left[\frac{1}{N}\sum f_iu_i^2 - \left(\frac{1}{N}\sum f_iu_i\right)^2\right] \times h^2$

h = difference between $x_i - x_{i-1} = 10$

Substituting values from the table, variance is,

$= \left[\frac{130}{70} - \left(\frac{22}{70}\right)^2\right] \times 100 = \left[\frac{13}{7} - \left(\frac{11}{35}\right)^2\right] \times 100$

= [1.857 - 0.099] $\times$ 100 = 175.8

and standard deviation = $\sqrt{Variance} = \sqrt{175.8}$ = 13.259

33. Let the equation of the circle be

$(x - h)^2 + (y - k)^2 = r^2$

If it passes through (7, 3) then

$(7 - h)^2 + (3 - k)^2 = (3)^2$ [$\because$ r = 3]

$49 + h^2 - 14h + 9 + k^2 - 6k = 9$

$h^2 + k^2 - 14h - 6k + 49 = 0$ ...(i)

If centre (h, k) lies on the line $y = x - 1$ then

$k = h - 1$ ...(ii)

Putting the value of k in eq. (i) we get

$h^2 + (h - 1)^2 - 14h - 6(h - 1) + 49 = 0$

$\Rightarrow h^2 + h^2 + 1 - 2h - 14h - 6h + 6 + 49 = 0$

$\Rightarrow 2h^2 - 22h + 56 = 0$

$\Rightarrow h^2 - 11h + 28 = 0$

$\Rightarrow h^2 - 7h - 4h + 28 = 0$

$\Rightarrow h(h - 7) - 4(h - 7) = 0$

$\Rightarrow (h - 4)(h - 7) = 0$

$\therefore h = 4, h = 7$

From eq. (ii) we get $k = 4 - 1 = 3$ and $k = 7 - 1 = 6$.

So, the centers are (4, 3) and (7, 6).

$\therefore$ Equation of the circle with centre (4, 3) and radius 3

$(x - 4)^2 + (y - 3)^2 = 9$

$x^2 + 16 - 8x + y^2 + 9 - 6y = 9$

$\Rightarrow x^2 + y^2 - 8x - 6y + 16 = 0$

Equation of the circle with centre (7, 6) and radius 3

$(x - 7)^2 + (y - 6)^2 = 9$

$\Rightarrow x^2 + 49 - 14x + y^2 + 36 - 12y = 9$

$\Rightarrow x^2 + y^2 - 14x - 12y + 76 = 0$

Hence, the required equations are

$x^2 + y^2 - 8x - 6y + 16 = 0$

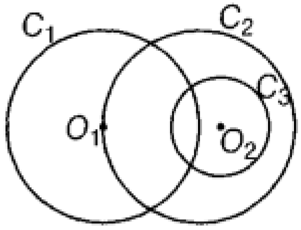
and $x^2 + y^2 - 14x - 12y + 76 = 0$.

OR

We have to find the equation of circle (C_2) which passes through the centre of circle (C_1) and is concentric with circle (C_3).

We have, equation of circle (C_1),

$x^2 + y^2 + 8x + 10y - 7 = 0$ ...(i)



On comparing it with $x^2 + y^2 + 2gx + 2fy + c = 0$, we get

$g = 4, f = 5$ and $c = -7$

$\therefore$ Centre of C_1 is $O_1 = (-g, -f)$

$O_1 = (-4, -5)$

Now, equation of circle (C_2) which is concentric with given circle (C_3) having equation $2x^2 + 2y^2 - 8x - 12y - 9 = 0$ is

$2x^2 + 2y^2 - 8x - 12y + k = 0$ ...(ii)

Since, circle (C_2) passes through O_1 (- 4, - 5)

$\therefore 2(-4)^2 + 2(-5)^2 - 8(-4) - 12(-5) + k = 0$

$\Rightarrow 32 + 50 + 32 + 60 + k = 0$

$\Rightarrow k = -174$

On putting the value of k in Eq. (ii), we get

$2x^2 + 2y^2 - 8x - 12y - 174 = 0$

$\Rightarrow x^2 + y^2 - 4x - 6y - 87 = 0$ [dividing both sides by 2]

which is required equation of circle (C_2).

34. The given system of linear inequalities is

$-2 - \frac{x}{4} \geq \frac{1+x}{3}$ ... (i)

and $3 - x < 4(x - 3)$ ... (ii)

From inequality (i), we get

$-2 - \frac{x}{4} \geq \frac{1+x}{3}$

$\Rightarrow -24 - 3x \geq 4 + 4x$ [multiplying both sides by 12]

$\Rightarrow -24 - 3x - 4 \geq 4 + 4x - 4$ [subtracting 4 from both sides]

$\Rightarrow -28 - 3x \geq 4x$

$\Rightarrow -28 - 3x + 3x \geq 4x + 3x$ [adding 3x on both sides]

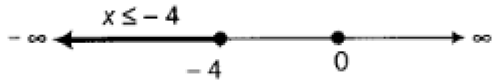
$\Rightarrow -28 \geq 7x$

$\Rightarrow -\frac{28}{7} \geq \frac{7x}{7}$ [dividing both sides by 7]

$\Rightarrow -4 \geq x$ or $x \leq -4$ ... (iii)

Thus, any value of x less than or equal to - 4 satisfied the inequality.

So, solution set is $x \in (-\infty, -4]$



From inequality (ii), we get

$3 - x < 4(x - 3)$

$\Rightarrow 3 - x < 4x - 12$

$\Rightarrow 3 - x + 12 < 4x - 12 + 12$ [adding 12 on both sides]

$\Rightarrow 15 - x < 4x$

$\Rightarrow 15 - x + x < 4x + x$ [adding x on both sides]

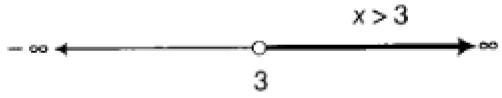
$\Rightarrow 15 < 5x$

$\Rightarrow 3 < x$ [dividing both sides by 3]

or $x > 3$ ... (iv)

Thus, any value of x greater than 3 satisfies the inequality.

So, the solution set is $x \in (3, \infty)$



The solution set of inequalities (i) and (ii) are represented graphically on number line as given below:

As no region is common, hence the given system has no solution.

35. We know,

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \sin^2 x$

$\cos^2 x = 1 - \left(\frac{1}{4}\right)^2$ ... [$\because \sin x = \frac{1}{4}$]

$\cos^2 x = 1 - \frac{1}{16} = \frac{16-1}{16} = \frac{15}{16}$

$\cos x = \pm\frac{\sqrt{15}}{4}$

Since, $x \in \left(\frac{\pi}{2}, \pi\right)$

$\Rightarrow$ cos x will be negative in second quadrant

So, $\cos x = -\frac{\sqrt{15}}{4}$

We know,

$\cos 2x = 2\cos^2 x - 1$

cos x = 2 $\cos^2\frac{x}{2}$ -1

$-\frac{\sqrt{15}}{4}$ = 2 $\cos^2\frac{x}{2}$ -1 ... [∵ cos x = $-\frac{\sqrt{15}}{4}$]

2 $\cos^2\frac{x}{2}$ = $-\frac{\sqrt{15}}{4}$ +1 = $\frac{-\sqrt{15}+4}{4}$

$\cos^2\frac{x}{2}$ = $\frac{-\sqrt{15}+4}{8}$

$\cos\frac{x}{2} = \pm\sqrt{\frac{-\sqrt{15}+4}{8}}$

Since, x ∈ $\left(\frac{\pi}{2}, \pi\right) \Rightarrow \frac{x}{2} \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$\cos\frac{x}{2}$ will be positive in first quadrant

So, $\cos\frac{x}{2} = \sqrt{\frac{-\sqrt{15}+4}{8}}$

We know,

cos 2x = 1 - 2 $\sin^2$ x

cos x = 1 - 2 $\sin^2\frac{x}{2}$ ... [∵ cos x = $-\frac{\sqrt{15}}{4}$]

$-\frac{\sqrt{15}}{4}$ =1 - 2 $\sin^2\frac{x}{2}$

2 $\sin^2\frac{x}{2}$ = $\frac{\sqrt{15}}{4}$ +1 = $\frac{\sqrt{15}+4}{4}$

$\sin^2\frac{x}{2} = \frac{\sqrt{15}+4}{8} = \pm\sqrt{\frac{\sqrt{15}+4}{8}}$

Since, x ∈ $\left(\frac{\pi}{2}, \pi\right) \Rightarrow \frac{x}{2} \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$\sin\frac{x}{2}$ will be positive in first quadrant

So, $\sin\frac{x}{2} = \sqrt{\frac{\sqrt{15}+4}{8}}$

We know,

$$\tan\frac{x}{2} = \frac{\sqrt{\frac{\sqrt{15}+4}{8}}}{\sqrt{\frac{-\sqrt{15}+4}{8}}}$$

$$\tan\frac{x}{2} = \sqrt{\frac{\sqrt{15}+4}{8} \times \frac{8}{-\sqrt{15}+4}}$$

$$\tan\frac{x}{2} = \sqrt{\frac{\sqrt{15}+4}{-\sqrt{15}+4}}$$

On rationalising:

$$\tan\frac{x}{2} = \sqrt{\frac{4+\sqrt{15}}{4-\sqrt{15}} \times \frac{4+\sqrt{15}}{4+\sqrt{15}}}$$

$\tan\frac{x}{2} = \sqrt{\frac{(4+\sqrt{15})^2}{4^2-(\sqrt{15})^2}}$ ... {∵ (a + b)(a - b) = a^2 - b^2}

$\tan\frac{x}{2} = \sqrt{\frac{(4+\sqrt{15})^2}{16-15}} = \sqrt{\frac{(4+\sqrt{15})^2}{1}}$ = 4 + $\sqrt{15}$

Hence, values of $\cos\frac{x}{2}$, $\sin\frac{x}{2}$ and $\tan\frac{x}{2}$ are $\sqrt{\frac{-\sqrt{15}+4}{8}}$, $\sqrt{\frac{\sqrt{15}+4}{8}}$ and 4 + $\sqrt{15}$ respectively

OR

We have to prove cot x + cot$\left(\frac{\pi}{3}+x\right)$ + cot$\left(\frac{2\pi}{3}+x\right)$ = 3 cot 3x.

LHS = cot x + cot$\left(\frac{\pi}{3}+x\right)$ + cot$\left(\frac{2\pi}{3}+x\right)$

We know,

cot$\left(\frac{2\pi}{3}+x\right)$ = cot$\left(\pi-\left(\frac{\pi}{3}-x\right)\right)$ = -cot$\left(\frac{\pi}{3}-x\right)$ ... (as - cotθ = cot (180^o - θ)

Hence the above LHS becomes

= cot x + cot$\left(\frac{\pi}{3}+x\right)$ - cot$\left(\frac{\pi}{3}-x\right)$

$= \frac{1}{\tan x} + \frac{1}{\tan\left(\frac{\pi}{3}+x\right)} - \frac{1}{\tan\left(\frac{\pi}{3}-x\right)}$

$= \frac{1}{\tan x} + \left(\frac{1-\tan x\tan\frac{\pi}{3}}{\tan\frac{\pi}{3}+\tan x}\right) - \left(\frac{1+\tan x\tan\frac{\pi}{3}}{\tan\frac{\pi}{3}-\tan x}\right)$ ... [∵ tan(A + B) = $\left(\frac{\tan A+\tan B}{1-\tan A\tan B}\right)$ and tan(A - B) = $\left(\frac{\tan A-\tan B}{1+\tan A\tan B}\right)$]

$= \frac{1}{\tan x} + \left(\frac{1-\sqrt{3}\tan x}{\sqrt{3}+\tan x}\right) - \left(\frac{1+\sqrt{3}\tan x}{\sqrt{3}-\tan x}\right)$

$= \frac{1}{\tan x} + \left(\frac{(1-\sqrt{3}\tan x)(\sqrt{3}-\tan x)-(1+\sqrt{3}\tan x)(\sqrt{3}+\tan x)}{(\sqrt{3}+\tan x)(\sqrt{3}-\tan x)}\right)$

$= \frac{1}{\tan x} + \left(\frac{(\sqrt{3}-\tan x-3\tan x+\sqrt{3}\tan^2 x)-(\sqrt{3}+3\tan x+\tan x+\sqrt{3}\tan^2 x)}{(3-\tan^2 x)}\right)$

$= \frac{1}{\tan x} + \left(\frac{(0-4\tan x-4\tan x+0)}{(3-\tan^2 x)}\right)$

$= \frac{1}{\tan x} - \left(\frac{8\tan x}{((3-\tan^2 x))}\right)$

$= \left(\frac{(3-\tan^2 x)-8\tan^2 x}{\tan x(3-\tan^2 x)}\right) = \left(\frac{(3-\tan^2 x)-8\tan^2 x}{\tan x(3-\tan^2 x)}\right)$

$= 3\left(\frac{1-3\tan^2 x}{(3\tan x-\tan^3 x)}\right)$

$= 3 \times \frac{1}{\tan 3x}$ ... (as tan 3x = $\frac{3\tan x-\tan^3 x}{1-3\tan^2 x}$)

= cot 3x

LHS = RHS

Hence proved.

Section E

36. i. pots in first row is 2

pots in second row is 4 and

pots in third row is 8 and so on ...

which is in GP with first term = a = 2 and common ratio = r = 2

ii. a = 2, r = 2, S_n = 510

$S_n = \frac{a(1-r^n)}{1-r}$

$\Rightarrow 510 = \frac{2((2)^n-1)}{2-1}$

$\Rightarrow \frac{510}{2} = (2)^n - 1$

$\Rightarrow 255 + 1 = (2)^n$

$\Rightarrow 2^8 = (2)^n$

$\Rightarrow$ n = 8

The total number of rows formed in this arrangement = 8

iii. a = 2, r = 2, n = 10

$S_n = \frac{a(1-r^n)}{1-r}$

$\Rightarrow S_{10} = \frac{2\left((2)^{10}-1\right)}{2-1}$

$\Rightarrow S_{10} = 2(1024 - 1) = 2 \times 1023 = 2046$

OR

a = 2, r = 2, n = 8

$S_n = \frac{a(1-r^n)}{1-r}$

$\Rightarrow S_8 = \frac{2\left((2)^8-1\right)}{2-1}$

$\Rightarrow S_8 = 2(256 - 1) = 2 \times 255 = 510$

37. i. Total number of possible outcomes = ${}^{52}C_4$

We know that there are 12 face cards

$\therefore$ Number of favourable outcomes = ${}^{12}C_4$

$\therefore$ Required probability $= \frac{{}^{12}C_4}{{}^{52}C_4}$

ii. Total number of possible outcomes = ${}^{52}C_4$

We know that there are 26 red and 26 black cards.

$\therefore$ Number of favourable outcomes = ${}^{26}C_2 \times {}^{26}C_2$

$\therefore$ Required probability $= \frac{\left({}^{26}C_2\right)^2}{{}^{52}C_4}$

iii. Total number of possible outcomes = ${}^{52}C_4$

$\therefore$ Number of favourable outcomes = $\left({}^{13}C_1\right)^4$

$\therefore$ Required probability = $\frac{(13)^4}{{}^{52}C_4}$

OR

Total number of possible outcomes = ${}^{52}C_4$

In playing cards there are 4 king and 4 jack cards.

$\because$ Number of favourable outcomes = $\left({}^{4}C_2 \times {}^{4}C_2\right)$

= 6 × 6 = 36

$\therefore$ Required probability = $\frac{36}{{}^{52}C_4}$

38. i. The intersecting of two sets A and C is A ∩ C $\neq \phi$

The intersecting of two sets A and C is = {3,5,7}

ii. Two sets A and B to be disjoint are A ∩ B = ϕ

iii. A ∩ C = {1, 3, 5, 7, 9} ∩ {2, 3, 5, 7, 11}

= {3, 5, 7}

OR

A ∩ B = {1, 3, 5, 7, 9} ∩ {2, 4, 6, 8}

= ϕ